MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

PCL0016 – CALCULUS

(ALL Sections/Groups)

01 JUNE 2017
9:00 A.M - 11:00 A.M.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of THREE pages including cover page with SIX questions.

2. Attempt **ALL SIX** questions and the distribution of the marks for each question is given. All answers must be written to the nearest thousandth or its simplest form.

3. Please write all your answers in the Answer Booklet provided.

Question 1 (20 marks)

(a) Evaluate the following limits, if they exist.

(i) $\lim_{x\to 5} 8(x-1)(2-x)$ [2 marks]

(ii) $\lim_{x\to\infty} \dfrac{\sqrt{x^5+2}}{x^3+1}$ [3 marks]

(iii) $\lim_{\theta\to 0} \dfrac{\sin^3\theta}{\theta(1-\cos\theta)}$ [5 marks]

(b) Find the limit $\lim_{x\to 1}[f(x)-g(x)]$, given the following functions:

$f(x)=5$ and $g(x)=\dfrac{-2x^2+x+1}{1-x}$ [5 marks]

(c) For what values of a is $h(x)$ continuous at every x?

$$h(x)=\begin{cases} a^2x^3+ax^2 & , \quad x<1 \\ x+5 & , \quad x\geq 1 \end{cases}$$ [5 marks]

Question 2 (15 marks)

(a) Given $\cos x - y^2 = x$. Find $y'(0)$ using implicit differentiation. [5 marks]

(b) Find $\dfrac{dy}{dx}$ of the following function using logarithmic differentiation.

$$y=\frac{\left(3x^2-1\right)^5\sqrt{x}}{(x+1)^8}$$ [5 marks]

(c) Use local linear approximation to estimate the value of $e^{0.022}$. [5 marks]

Continued...

Question 3 (15 marks)

(a) Find the absolute minimum and maximum values of the function below on the interval $[-0.5,\ 1.5]$.

$$f(x) = \frac{x}{x^2+1}$$ [8 marks]

(b) An open box (without a lid) is made from a 20cm and 40cm cardboard by cutting out squares of equal size from the four corners and bending the sides. Each square has a length of x cm. The volume of the box is given as:

$$V = x(20-2x)(40-2x)$$

Find x to obtain a box with the largest volume. Use second derivative test to proof that the proposed x gives the largest volume.

[7 marks]

Question 4 (15 marks)

(a) Evaluate $\int_0^{\pi} 3x^2 \sin(2x)\, dx$. [7 marks]

(b) Use integration by partial fraction to find $\int \frac{1}{x(x^2+1)}\ dx$. [8 marks]

Question 5 (15 marks)

A region is bounded by $y = x+1$, $y = 0$, $x = 0$ and $x = 4$.

(a) Sketch and shade the bounded region. [3 marks]

(b) The shaded region above revolves about the y-axis. Calculate the volume using cylindrical shell method.

[5 marks]

(c) Prove that the volume is the same using disk/washer method. [7 marks]

Question 6 (20 marks)

(a) Solve $\frac{dy}{dx} = e^{-3y}x^2$ with initial condition $y(2) = 0$. [6 marks]

(b) Solve the differential equations below:

(i) $y'' + y' - 6y = 0$, given $y(0) = 0$ and $y'(0) = 10$. [7 marks]

(ii) $y'' + 9y = 0$, given $y(0) = 2$ and $y'(\frac{\pi}{3}) = -3$. [7 marks]

End of Paper